Evaluation of measurement data obtained from fluid flow measurement

Appendix 3 to the guidelines for the measurement regulations

Table of Contents

Introduction

This appendix sets out the principles for evaluating measurement data in connection with measuring fluid flow. The purpose of this appendix is to provide a more detailed explanation of the requirements for calibration and verification of flow meters. The appendix is mainly based on ISO 5168 (cf. Appendix 2).

Measurement error and uncertainty

Mean measurement error

The mean measurement error is determined at each flow rate as an arithmetic mean, given by the following equation:

$$\bar{E} = \frac{\sum_{i=1}^{n} E_i}{n} \tag{1}$$

where

E_i is the *i*-th relative measurement error,
$\bar{E}$ is the mean relative measurement error, and
n is the number of measurements of the same quantity at a single flow rate.

The *i*-th relative measurement error is calculated using the following equation:

$$E_i = \frac{Q_{ind} - Q_{ref}}{Q_{ref}} \tag{2}$$

where Q_{ind} is the flow rate measured by a meter during tests and Q_{ref} is the flow rate measured by the measurement standard, corrected for any thermodynamic differences in the fluid at the meter during tests and at the measurement standard.

Random uncertainty in the mean measurement error

The random uncertainty in the mean measurement error is determined at each flow rate as a statistical uncertainty calculated from a series of n measurements, as given by the following equation:

$$U_{AM-E} = \frac{U_{AS-E}}{\sqrt{n}} \tag{3}$$

where U_{AS-E} is the random uncertainty in single measurements (repeatability of a meter), which is determined at each flow rate and calculated using the following equation:

$$U_{AS-E} = t_{95,n-1} \cdot \sqrt{\frac{\sum_{i=1}^{n} (E_i - \bar{E})^2}{n-1}} = t_{95,n-1} \cdot s \tag{4}$$

and where s is the experimental standard deviation (for a series of n measurements of the same quantity), $t_{95,\,n-1}$ is the Student's t-distribution factor for a 95 % confidence level and $n - 1$ degrees of freedom.

It follows from e.g. API MPMS 13.3 that the experimental standard deviation of a series of n measurements can be approximated by the following equation:

$$s \cong \frac{w_{(n)}}{d_{(n)}} \quad (5)$$

where $w_{(n)}$ is the range, the difference between the maximum and minimum values for a set of measurement data, and $d_{(n)}$ is a conversion factor for estimating standard deviations for n measurements. Values for $d_{(n)}$ can e.g. be found in Table E.1 in API MPMS 13.3. They may also be calculated based on the expected values of the ratio $w_{(n)}/s$ for normally distributed random individual measurements. By combining the above equations, an approximation of the uncertainty of the mean value of a series of n measurements can be expressed by the following equation:

$$U_{AM-E} \cong \frac{t_{95,n-1} \cdot w_{(n)}}{\sqrt{n} \cdot d_{(n)}} \quad (6)$$

Example: $w_{(5)} = 0{,}05\ \% \rightarrow U_{AM-E} = 0{,}027\ \%$.

Combined uncertainty in the mean measurement error

The combined uncertainty in the mean measurement error is determined at each flow rate using the following equation:

$$U_{CM-E} = \sqrt{{U_{AM-E}}^2 + {U_{CMC}}^2} \quad (7)$$

where U_{CMC} is the combined uncertainty of the calibration setup (CMC is an abbreviation for *Calibration and Measurement Capability*), including the uncertainty of the measurement standard. Since U_{AM-E} can be reduced by increasing the number of measurements in the series, U_{CM-E} is often only marginally greater than U_{CMC}.

Evaluation of measurement error

According to OIML R137:2012 and ISO 17089:2019, a measurement error can be considered within a specified error limit (MPE) if the mean measurement error is within the acceptance limits in Table 1.

Table 1. Acceptance limits for measurement errors

Mean measurement error (deviation in indication)	Combined uncertainty of mean value	Acceptance limit
$\overline{\boldsymbol{E}}$	$U_{CM-E} < \frac{1}{3} \cdot MPE$	MPE
	$U_{CM-E} \in [1/3 \cdot MPE, MPE]$	$\frac{4}{3} \cdot MPE - U_{CM-E}$
	$U_{CM-E} > MPE$	Undefined

If $U_{CM-E} > MPE$, the combined uncertainty of the mean value is too high to verify compliance with the MPE requirement.

The method and principle described above are further elaborated in JCGM 106:2012 and OIML G-19:2017. These documents may be useful when developing methods for evaluating measurement errors and establishing acceptance limits for calibration and verification (see e.g. Figure 7 in JCGM 106:2012).

Example: Evaluation of deviation between indications from two meters in series

In this example, it is assumed that the methodology described above can be applied using two meters, where verification of the requirement for instrumental measurement uncertainty is performed by measuring the deviation between indications from the two meters. Typically, the meter functioning as the primary meter (meter A) is the one to be verified, while the other meter (meter B) serves as the reference. At a given flow rate, the *i*-th relative measurement error is determined by the following equation:

$$E_i = \frac{Q_A - Q_B}{Q_B} \tag{8}$$

where Q_A is the flow rate measured by meter A and Q_B is the flow rate measured by meter B, corrected for thermodynamic differences in the fluid at meter A and B. The mean measurement error is then determined by the:

$$\bar{E} = \frac{\sum_{i=1}^{n} E_i}{n} \tag{9}$$

The combined uncertainty of the mean (relative) measurement error can be determined by the following equation:

$$U_{CM-E} = \sqrt{{U_{AM-E}}^2 + {U_B}^2} \tag{10}$$

where U_B is the instrumental measurement uncertainty of meter B (value given in the uncertainty budget or on the certificate), excluding uncertainty components that are fully correlated between the two meters. It can then be presumed that meter A meets the uncertainty limit requirement for instrumental measurement uncertainty, U_g, if the mean measurement error is within the acceptance limits in Table 2.

Table 2. Acceptance limits for deviations in indications from two meters

Mean measurement errors (deviation between indications)	Combined uncertainty of mean value	Acceptance limit
$\bar{\boldsymbol{E}}$	$U_{CM-E} < \frac{1}{3} \cdot U_g$	U_g
	$U_{CM-E} \in [1/3 \cdot U_g, U_g]$	$\frac{4}{3} \cdot U_g - U_{CM-E}$
	$U_{CM-E} > U_g$	Undefined

For example, an uncertainty limit for instrumental measurement uncertainty of $U_g = 0.20$ % for an oil meter in a delivery measuring system (cf. the guideline to Section 28), and a combined uncertainty in the mean measurement errors of $U_{CM-E} = 0.15$ %, would result in an acceptance limit of 0.12 % for the mean measurement error.

Calibration factor (*K*-factor) and uncertainty

Mean *K*-factor

The mean *K* factor is determined at each flow rate as an arithmetic mean, given by the following equation:

$$\overline{K} = \frac{\sum_{i=1}^{n} K_i}{n} \tag{11}$$

where

K_i is the *i*-th absolute *K*-factor,
$\overline{K}$ is the mean absolute *K*-factor, and
n is the number of measurements of the same quantity at a single flow rate.

Random uncertainty in the mean *K*-factor

The random uncertainty in the mean *K*-factor is determined at each flow rate as a statistical uncertainty calculated from a series of n measurements, given by the following equation:

$$U_{AM-K} = \frac{U_{AS-K}}{\sqrt{n}} \tag{12}$$

where U_{AS-K} is the random uncertainty in single measurements (repeatability of a meter) determined by the following equation:

$$U_{AS-K} = \frac{t_{95,n-1}}{\overline{K}} \cdot \sqrt{\frac{\sum_{i=1}^{n}(K_i - \overline{K})^2}{n-1}} = t_{95,n-1} \cdot \frac{s}{\overline{K}} \tag{13}$$

An approximation for U_{AS-K} can be found in the substitution $s \cong w_{(n)}/d_{(n)}$ (cf. Equation (5)).

Example: $w_{(5)} = 0.05\ \% \rightarrow U_{AM-K} = 0.027\%$.

Combined uncertainty in the mean *K* factor

The combined uncertainty in the mean *K*-factor is determined at each flow rate using the following equation:

$$U_{CM-K} = \sqrt{{U_{AM-K}}^2 + {U_{CMC}}^2} \tag{14}$$

Linearity over the flow rate range

Linearity, expressed by (relative) measurement error, is determined as:

$$\overline{E}_{MAX} - \overline{E}_{MIN}$$

where $\overline{E}_{MAX}$ and $\overline{E}_{MIN}$ are the highest and lowest values of the mean measurement error, respectively, over the flow rate range.

Linearity, expressed by the (absolute) *K* factor, is determined as:

$$\frac{\overline{K}_{MAX} - \overline{K}_{MIN}}{\overline{\overline{K}}}$$

where

$$\overline{\overline{K}} = \frac{\sum_{j=1}^{m} \overline{K_J}}{m} \tag{15}$$

and m is the number of measurements of the same quantity over the flow rate range. The calibration factors $\overline{K}_{MAX}$ and $\overline{K}_{MIN}$ are the highest and lowest mean *K*-factor, respectively, over the flow rate range.